

## **Accurate Measurement of the Thermal Conductivity and Thermal Diffusivity of Toluene and *n*-Heptane<sup>1</sup>**

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Accurate and simultaneous measurements of the thermal conductivity and thermal diffusivity of toluene and *n*-heptane were made with an improved transient hot-wire method by using a transfer function having a feedback loop, in the temperature range of 0 to 45°C at atmospheric pressure. The accuracy of the empirical equations as a function of temperature is estimated to be 0.4 to 0.5% for the thermal conductivity and about 4% for the thermal diffusivity.

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**KEY WORDS:** *n*-heptane; reference material; specific heat capacity; thermal conductivity; thermal diffusivity; toluene; transient hot-wire method.

### **1. INTRODUCTION**

The transient hot-wire (THW) method is a well-known method to measure thermal conductivity and thermal diffusivity simultaneously, although the application to thermal diffusivity is considered to be lacking in both accuracy and stability. The author has already reported investigations on this method [1-3]. In this paper, data on the thermal conductivity and thermal diffusivity of toluene and *n*-heptane in the temperature range 0 to 45°C at atmospheric pressure are presented.

### **2. MEASUREMENT PROCEDURE**

In the THW method, the heat generated in the wire changes depending on the circuit constants and temperature rise of the wire for the elapsed time  $t > 0$  as follows:

$$q(t) = Q_0(1 + A \Delta T(t) + B \Delta T^2(t) + \dots) \quad (1)$$

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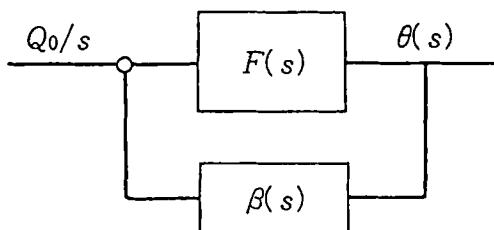
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where  $Q_0$  is the initial heating rate of the wire at  $t=0$ ,  $\Delta T(t)$  is the temperature rise of the wire at  $t$ , and  $A$  and  $B$  are fixed by the experimental conditions such as the circuit configuration including the kind of its power source (either voltage or current).

Therefore, the model of the THW method can be described in terms of a feedback loop with coefficients  $A$  and  $B$  as shown in Fig. 1. The temperature rise of the wire,  $\Delta T(t)$ , can be represented as [3]

$$\begin{aligned}
 \Delta T(t) = & \left( \frac{Q_0}{4\pi\lambda} \right) \left\{ \ln \frac{4\kappa t}{a^2 C} + \frac{a^2}{2\kappa t} \left( \frac{k-1}{k} \ln \frac{4\kappa t}{a^2 C} + 1 \right) \right. \\
 & - \frac{a^2}{4\kappa_W t} + \frac{\lambda}{2\lambda_W} + O \left[ \frac{a^2}{4\kappa t} \right]^2 \Big\} \\
 & + A \left( \frac{Q_0}{4\pi\lambda} \right)^2 \left\{ \left( \ln \frac{4\kappa t}{a^2 C} \right)^2 + \frac{a^2}{\kappa t} \left( \ln \frac{4\kappa t}{a^2 C} + 1 \right) - \frac{\pi^2}{6} + \dots \right\} \\
 & + A^2 \left( \frac{Q_0}{4\pi\lambda} \right)^3 \left\{ \left( \ln \frac{4\kappa t}{a^2 C} \right)^3 - \frac{\pi^2}{2} \ln \frac{4\kappa t}{a^2 C} + 2\zeta(3) + \dots \right\} \\
 & + B \left( \frac{Q_0}{4\pi\lambda} \right)^3 \left\{ \left( \ln \frac{4\kappa t}{a^2 C} \right)^3 - \frac{\pi^2}{3} \ln \frac{4\kappa t}{a^2 C} + 2\zeta(3) + \dots \right\} \\
 & + \delta T_{\text{rad}}(t) + \delta T_C(t) + \delta T_{\text{pot}}(t) \tag{2}
 \end{aligned}$$

where  $\lambda$  and  $\kappa$  are the thermal conductivity and thermal diffusivity of the sample liquid, and  $\lambda_W$  and  $\kappa_W$  those of the hot-wire material (platinum),  $a$  is the radius of the hot wire ( $\cong 10 \mu\text{m}$ ),  $k$  is the ratio of the volume heat



**Fig. 1.** Equivalent diagram for the THW method with a feedback loop.  $F(s)$  is the ideal transfer function involving both  $\lambda$  and  $\kappa$  as parameters.  $\beta(s)$  is a feedback transfer function.  $Q_0/s$  is a step input of heat generation in the wire, and  $\theta(s)$  is the corresponding temperature rise.

capacities of sample and wire materials ( $cp/(cp)_w$ ) ( $c$ , specific heat capacity;  $\rho$ , density),  $C = \exp(\gamma)$  ( $\gamma$ , Euler's constant),  $\zeta(m=3)$  is the Riemann zeta function, and then  $\delta T_{\text{rad}}(t)$  is the radiative heat transfer correction,  $\delta T_c(t)$  the wall effect correction (in which the concept of the reflectivity of a thermal wave at wall is introduced) [2, 3], and  $\delta T_{\text{pot}}(t)$  the potential leads correction.

In this study, in addition to the terms in Eq. (2), other corrections or uncertainty terms were investigated [3]:

- (a) instantaneous assignment of data-acquisition time,
- (b) nonlinear relationship between the bridge output signal and the wire-resistance variation,
- (c) temperature dependence of the derivative of the wire resistance vs temperature.

Two hot-wire sensors (four-terminal resistance) were made of platinum wire 20  $\mu\text{m}$  in diameter, which were set in the glass tube and installed as an arm of the double bridge. One hot wire (designated the AD cell) was constructed with a stainless frame, and the other (CF cell) with the frame made of machinable ceramics and Teflon. The wires were tightened through the soft spring coils (two turns) at each end, made of silver wire 0.2 mm in diameter. The potential leads were connected to the hot wire with platinum wires 15  $\mu\text{m}$  in diameter. The effective diameter of the platinum wire  $2a$  was determined from the measured mass, length, and density [3].

The resistance-temperature relationship for the platinum wire was measured with a platinum resistance thermometer (Chino R800-1) and the automatic bridge (ASL F18) in the temperature range  $-0.5$  to  $45^\circ\text{C}$  with a controlled water bath with a stability better than 1 mK, and the bath temperature  $T_B$  was measured with an accuracy better than 1 mK with a quartz thermometer (HP-2804A) calibrated by the same Pt thermometer.

### 3. RESULTS AND EVALUATIONS OF UNCERTAINTY

#### 3.1. Results

The present work for toluene and *n*-heptane was repeated with two AD and CF cells at various bath temperatures between 0 and  $45^\circ\text{C}$  with a fixed configuration using a constant voltage source but various heating rates.

The substances used in the experiments were special reagents (Kishida Chemical Co., Japan) for spectroscopy, and they were not analyzed further.

**Table I.** Experimental Data for Toluene

Datum	$T_b$ (°C)	$T_m$ (°C)	$\lambda$ (W·m <sup>-1</sup> ·K <sup>-1</sup> )	$\kappa$ (10 <sup>-8</sup> m <sup>2</sup> ·s <sup>-1</sup> )	$Q_0$ (W·m <sup>-1</sup> )
TVA000	21.732	22.176	0.13215	9.212	0.1264
TVA001	21.732	22.427	0.13189	9.159	0.1974
TVA002	21.732	23.274	0.13151	9.074	0.4367
TVA003	21.732	24.742	0.13112	8.995	0.8491
TVD000	21.732	22.178	0.13202	9.132	0.1268
TVD001	21.732	22.426	0.13198	9.155	0.1971
TVD002	21.732	23.272	0.13154	9.054	0.4362
TVD003	21.732	24.755	0.13106	8.932	0.8528
TVC000	21.732	22.187	0.13221	9.061	0.1298
TVC001	21.732	22.422	0.13182	8.976	0.1965
TVC002	21.732	23.269	0.13180	9.088	0.4436
TVC003	21.732	24.812	0.13144	9.063	0.8689
TVF000	21.732	22.203	0.13185	9.194	0.1335
TVF001	21.732	22.458	0.13186	9.288	0.2053
TVF002	21.732	23.351	0.13138	9.118	0.4573
TVF003	21.732	24.884	0.13091	9.019	0.8861
TTC000	21.732	22.423	0.13195	9.254	0.1958
TTC001	21.732	23.273	0.13171	9.205	0.4357
TTC002	21.732	24.765	0.13128	9.113	0.8544
TTF000	21.732	22.457	0.13183	9.204	0.2053
TTF001	21.732	23.336	0.13139	9.149	0.4530
TTF002	21.732	24.880	0.13093	9.085	0.8843
TVA100	2.406	2.826	0.13779	9.746	0.1233
TVA101	2.406	3.053	0.13773	9.701	0.1899
TVA102	2.406	3.845	0.13742	9.660	0.4209
TVA103	2.406	5.233	0.13690	9.517	0.8238
TVA104	2.406	3.045	0.13770	9.719	0.1873
TVC100	2.406	3.056	0.13791	9.690	0.1909
TVC101	2.405	3.875	0.13743	9.663	0.4296
TVC102	2.407	5.292	0.13701	9.555	0.8405
TVD100	2.406	3.061	0.13777	9.707	0.1920
TVD101	2.406	3.861	0.13743	9.621	0.4259
TVD102	2.406	5.260	0.13699	9.535	0.8316
TVF100	2.405	3.087	0.13749	9.696	0.1995
TVF101	2.405	3.914	0.13710	9.631	0.4403
TVF102	2.405	5.366	0.13670	9.581	0.8595
TTC100	2.405	3.045	0.13787	9.771	0.1877
TTC101	2.405	3.851	0.13734	9.660	0.4227
TTC102	2.405	5.252	0.13685	9.498	0.8294
TTF100	2.405	3.079	0.13748	9.756	0.1972
TTF101	2.405	3.900	0.13709	9.654	0.4363
TTF102	2.405	5.326	0.13674	9.448	0.8510
TVA200	40.309	41.042	0.12575	8.675	0.2001
TVD200	40.310	41.062	0.12582	8.728	0.2052

Table I. (Continued)

Datum	$T_B$ (°C)	$T_m$ (°C)	$\lambda$ (W · m <sup>-1</sup> · K <sup>-1</sup> )	$\kappa$ (10 <sup>-8</sup> m <sup>2</sup> · s <sup>-1</sup> )	$Q_0$ (W · m <sup>-1</sup> )
TVA201	40.309	41.954	0.12544	8.643	0.4480
TVD201	40.310	41.951	0.12541	8.640	0.4468
TVA202	40.309	43.526	0.12495	8.537	0.8726
TVD202	40.311	43.545	0.12488	8.519	0.8771
TVC200	40.310	41.050	0.12642	8.608	0.2038
TVF200	40.310	41.067	0.12615	8.367	0.2090
TVC201	40.310	41.948	0.12617	8.292	0.4528
TVF201	40.311	41.994	0.12573	8.290	0.4638
TVC202	40.310	43.533	0.12581	8.197	0.8884
TVF202	40.311	43.635	0.12544	8.284	0.9116
TVC203	40.313	43.563	0.12577	8.381	0.8919
TVF203	40.313	43.630	0.12552	8.342	0.9087
TTC200	40.313	41.053	0.12667	8.717	0.2039
TTF200	40.311	41.059	0.12560	8.417	0.2057
TTC201	40.314	41.949	0.12597	8.213	0.4517
TTF201	40.312	41.990	0.12515	8.209	0.4611
TTC202	40.314	43.491	0.12601	8.318	0.8746
TTF202	40.312	43.622	0.12465	8.130	0.9046
TLA001	24.882	25.580	0.13006	9.044	0.1958
TLD001	24.883	25.588	0.13008	9.030	0.1977
TLA002	24.882	26.439	0.12972	8.965	0.4357
TLD002	24.883	26.448	0.12974	8.977	0.4378
TLA003	24.883	27.664	0.12938	8.931	0.7753
TLD003	24.884	27.682	0.12928	8.870	0.7803
TTC001	24.883	25.591	0.13039	9.026	0.1993
TLF001	24.884	25.618	0.12996	8.987	0.2060
TLC002	24.883	26.469	0.13009	9.016	0.4445
TLF002	24.884	26.507	0.12959	8.861	0.4544
TLC003	24.883	27.745	0.12981	9.001	0.7989
TLF003	24.883	27.787	0.12916	8.803	0.8095
TLA101	42.052	42.791	0.12510	8.514	0.2016
TLD101	42.053	42.796	0.12555	8.765	0.2022
TLA102	42.052	43.695	0.12481	8.496	0.4464
TLD102	42.053	43.705	0.12497	8.594	0.4487
TLA103	42.052	45.009	0.12454	8.488	0.8007
TLD103	42.053	45.027	0.12448	8.454	0.8053
TLC101	42.053	42.815	0.12569	8.606	0.2085
TLF101	42.053	42.836	0.12519	8.578	0.2135
TLC102	42.052	43.724	0.12543	8.590	0.4458
TLF102	42.053	43.772	0.12489	8.577	0.4668
TLC103	42.053	45.079	0.12494	8.566	0.8204
TLF103	42.053	45.150	0.12445	8.438	0.8382

Table II. Experimental Data for *n*-Heptane

Datum	$T_b$ (°C)	$T_m$ (°C)	$\lambda$ (W · m <sup>-1</sup> · K <sup>-1</sup> )	$\kappa$ (10 <sup>-8</sup> m <sup>2</sup> · s <sup>-1</sup> )	$Q_0$ (W · m <sup>-1</sup> )
KVA000	22.342	22.520	0.12252	8.215	0.0480
KVA001	22.342	22.950	0.12191	8.211	0.1630
KVA002	22.342	24.412	0.12111	7.898	0.5520
KVA003	22.342	25.545	0.12052	7.824	0.5620
KVD001	22.340	24.428	0.12060	7.857	0.5550
KVD002	22.339	25.555	0.12024	7.811	0.8520
KVC001	22.339	23.102	0.12191	8.098	0.2050
KVC002	22.339	24.000	0.12164	8.028	0.4440
KVC003	22.339	25.607	0.12108	7.949	0.8700
KVC004	22.340	22.810	0.12189	7.986	0.1270
KVF001	22.340	22.834	0.12163	8.013	0.1330
KVF002	22.341	24.050	0.12210	7.988	0.4590
KVF003	22.341	25.659	0.12146	7.894	0.8870
KVF004	22.341	22.955	0.12257	8.048	0.1660
KVC005	22.341	22.937	0.12322	8.052	0.1620
KVC006	22.341	23.221	0.12315	8.074	0.2380
KVC007	22.341	23.979	0.12287	8.050	0.4420
KVC008	22.341	25.551	0.12237	7.971	0.8630
KVA005	22.342	23.233	0.12167	7.979	0.2390
KVA006	22.342	25.536	0.12069	7.834	0.8510
KVD003	22.342	23.971	0.12138	7.931	0.4360
KVD004	22.342	25.535	0.12070	7.766	0.8510
KVD005	22.342	23.218	0.12171	7.955	0.2350
KVA100	-0.132	0.571	0.12871	8.381	0.2000
KVA101	-0.132	1.343	0.12833	8.353	0.4180
KVA102	-0.132	2.773	0.12778	8.280	0.8210
KVA103	-0.133	0.303	0.12876	8.361	0.1260
KVD100	-0.133	0.541	0.12879	8.438	0.1920
KVD101	-0.133	1.350	0.12845	8.333	0.4210
KVD102	-0.133	2.777	0.12784	8.211	0.8230
KVD103	-0.133	0.311	0.12887	8.340	0.1270
KVC100	-0.133	0.314	0.12980	8.645	0.1270
KVC101	-0.133	0.546	0.12951	8.525	0.1930
KVC102	-0.133	1.362	0.12914	8.474	0.4230
KVC103	-0.133	2.809	0.12867	8.411	0.8290
KVF100	-0.134	0.310	0.12910	8.571	0.1250
KVF101	-0.133	0.569	0.12892	8.524	0.1990
KVF102	-0.134	1.407	0.12852	8.483	0.4340
KVF103	-0.134	2.910	0.12787	8.353	0.8530
KVF104	-0.134	1.601	0.12872	8.578	0.4350
KVA200	12.542	12.996	0.12511	8.265	0.1250
KVA201	12.542	13.243	0.12484	8.212	0.1920
KVA202	12.542	14.108	0.12451	8.186	0.4270

Table II. (Continued)

Datum	$T_B$ (°C)	$T_m$ (°C)	$\lambda$ (W · m <sup>-1</sup> · K <sup>-1</sup> )	$\kappa$ (10 <sup>-8</sup> m <sup>2</sup> · s <sup>-1</sup> )	$Q_0$ (W · m <sup>-1</sup> )
KVA203	12.542	15.626	0.12386	8.070	0.8380
KVD200	12.543	12.995	0.12513	8.250	0.1240
KVD201	12.543	13.259	0.12504	8.282	0.1250
KVD202	12.543	14.116	0.12460	8.173	0.4290
KVD203	12.543	15.636	0.12393	8.019	0.8410
KVC200	12.542	12.995	0.12622	8.409	0.1250
KVC201	12.542	13.239	0.12604	8.334	0.1920
KVC202	12.543	14.113	0.12568	8.264	0.2730
KVC203	12.542	15.636	0.12515	8.173	0.8480
KVF200	12.543	13.026	0.12575	8.451	0.1330
KVF201	12.543	13.270	0.12542	8.352	0.2000
KVF202	12.543	14.172	0.12498	8.270	0.4460
KVF203	12.543	15.753	0.12428	8.126	0.8740
KVA300	35.378	35.884	0.11799	7.867	0.1320
KVA301	35.379	36.143	0.11770	7.774	0.2000
KVA302	35.379	37.081	0.11728	7.695	0.4430
KVA303	35.379	38.728	0.11661	7.552	0.8680
KVD300	35.380	35.868	0.11809	7.741	0.1280
KVD301	35.380	36.142	0.11790	7.726	0.2000
KVD302	35.380	37.084	0.11740	7.614	0.4440
KVD303	35.380	38.739	0.11673	7.499	0.8710
KVC300	35.380	35.876	0.11895	7.915	0.1310
KVC301	35.380	36.157	0.11873	7.967	0.2050
KVC302	35.380	37.094	0.11858	8.011	0.4510
KVC303	35.380	38.772	0.11823	7.948	0.8890
KVF300	35.381	35.884	0.11820	7.834	0.1320
KVF301	35.381	36.175	0.11843	8.114	0.2090
KVF302	35.381	37.156	0.11851	8.176	0.4650
KVF303	35.381	38.866	0.11833	8.161	0.9090
KVA400	27.004	27.477	0.12040	7.959	0.1250
KVA401	27.005	27.740	0.12038	7.956	0.1950
KVA402	27.005	28.657	0.11995	7.901	0.4370
KVA403	27.005	30.257	0.11928	7.771	0.8570
KVD400	27.006	27.475	0.12050	7.875	0.1250
KVD401	27.006	27.755	0.12047	7.940	0.1990
KVD402	27.006	28.662	0.11999	7.851	0.4390
KVD403	27.007	30.268	0.11933	7.718	0.8600
KVC400	27.005	27.504	0.12076	7.981	0.1330
KVC401	27.006	27.752	0.12090	8.074	0.1980
KVC402	27.005	28.689	0.12069	8.084	0.4470
KVC403	27.006	30.313	0.12028	8.072	0.8740
KVF400	27.007	27.510	0.12043	7.933	0.1340
KVF401	27.007	27.782	0.12108	8.297	0.2060

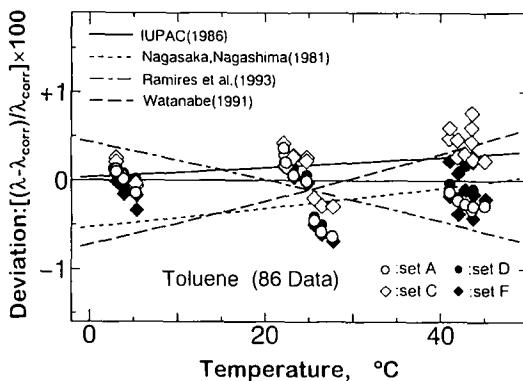
Table II. (Continued)

Datum	$T_B$ (°C)	$T_m$ (°C)	$\lambda$ (W·m <sup>-1</sup> ·K <sup>-1</sup> )	$\kappa$ (10 <sup>-8</sup> m <sup>2</sup> ·s <sup>-1</sup> )	$Q_0$ (W·m <sup>-1</sup> )
KVF402	27.007	28.737	0.12002	8.002	0.4570
KVF403	27.006	30.401	0.11988	8.038	0.8930
KVA500	44.269	44.780	0.11543	7.750	0.1310
KVA501	44.270	45.069	0.11507	7.563	0.2050
KVA502	44.270	46.025	0.11465	7.505	0.4480
KVA503	44.270	47.721	0.11401	7.395	0.8780
KVD500	44.276	44.796	0.11541	7.683	0.1330
KVD501	44.277	45.060	0.11521	7.568	0.2010
KVD502	44.276	46.035	0.11472	7.475	0.4490
KVD503	44.276	47.739	0.11404	7.329	0.8810
KVC500	44.270	44.782	0.11636	7.848	0.1320
KVC501	44.270	45.063	0.11592	7.678	0.2040
KVC502	44.271	46.060	0.11554	7.669	0.4580
KVC503	44.276	47.788	0.11517	7.638	0.8960
KVF500	44.276	44.795	0.11573	7.667	0.1330
KVF501	44.276	45.098	0.11558	7.725	0.2110
KVF502	44.276	46.114	0.11595	7.981	0.4690
KVF503	44.276	47.888	0.11511	7.781	0.9180
KVF504	44.276	47.888	0.11498	7.716	0.9180

Water (in comparison with other impurities) may greatly influence the property values. Therefore, it was removed by means of a molecular sieve, although it may not be perfect for toluene. The residual water labeled for the toluene was less than 0.03%, but for *n*-heptane, not labeled (it could be neglected). The filling procedure of the sample into the cell was by siphon, without exposure to the atmosphere as far as possible.

The measured data for toluene and *n*-heptane are listed in Tables I and II, respectively. Slight differences in the data obtained with the AD vs the CF cells may be observed [3], but they were all analyzed together. Figures 2 to 5 show the deviations of the measured data and the other formulas [2, 4-6] from a correlation as given by Eqs. (4) and (5) for  $\lambda$  and  $\kappa$ . Correlating analyses have been made for thermal conductivity [7], at temperature  $T_m$ , which is defined by the following equation, and for thermal diffusivity [8], at the initial (or bath) temperature  $T_B$ .

$$T_m = T_B + (1/2)\{\Delta T(t_i) + \Delta T(t_f)\} \quad (3)$$



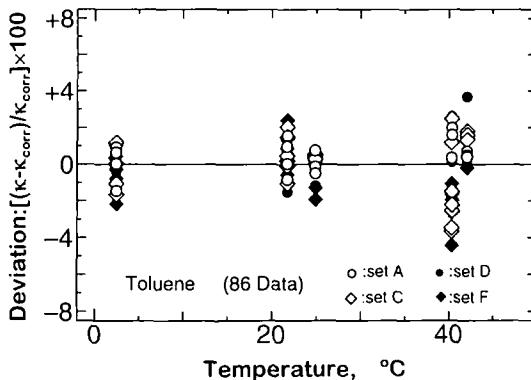
**Fig. 2.** Deviations of measured values of the thermal conductivity of toluene and other formulas [2, 4–6] from Eq. (4a).

where  $t_i$  and  $t_f$  are the elapsed time assigned to the initial and final acquisition data of one experiment, and nominally  $t_i = 0.03$  s and  $t_f = 1$  s in the present case.

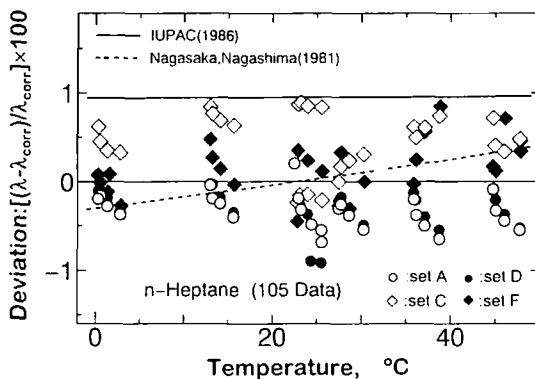
As a result, the empirical equations for  $\lambda$  (in  $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ ) and  $\kappa$  (in  $10^{-8} \text{ m}^2 \cdot \text{s}^{-1}$ ) in terms of  $T$  in K between 0 and 50°C are as follows.

### 3.1.1. Toluene (86 Data)

$$\lambda = 0.13855(08) - 3.070(27) \times 10^{-4}(T - 273.15) \quad (4a)$$



**Fig. 3.** Deviations of measured values of the thermal diffusivity of toluene from Eq. (4b).



**Fig. 4.** Deviations of measured values of the thermal conductivity of *n*-heptane and other formulas [4, 5] from Eq. (5a).

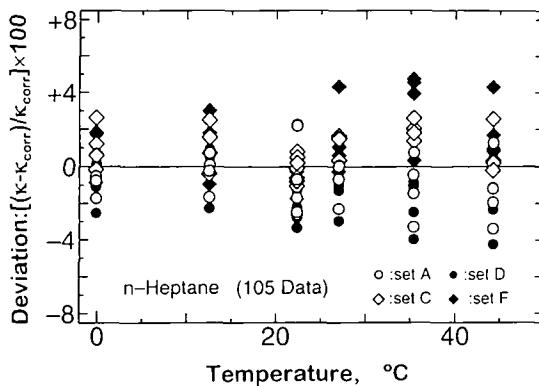
and the experimental standard deviation of the mean (i.e., of the equation) is given by the following equation (the uncertainty analysis is given in the ISO Guide [9]):

$$\sigma_\kappa \cong [(4 \times 10^{-5})^2 + (2.7 \times 10^{-6} \Delta\theta)^2]^{1/2} \quad (4a')$$

where  $\Delta\theta$  is  $T - 298.15$  K. For thermal diffusivity,

$$\kappa = 9.730(28) - 0.0303(10) \cdot (T - 273.15) \quad (4b)$$

$$\sigma_\kappa \cong [(0.014)^2 + (0.0010 \Delta\theta)^2]^{1/2} \quad (4b')$$



**Fig. 5.** Deviations of measured values of the thermal diffusivity of *n*-heptane from Eq. (5b).

### 3.1.2. *n*-Heptane (105 Data)

$$\lambda = 0.12909(10) - 3.029(36) \times 10^{-4}(T - 273.15) \quad (5a)$$

$$\sigma_\lambda \cong [(4 \times 10^{-5})^2 + (3.6 \times 10^{-6} \Delta \theta)^2]^{1/2} \quad (5a')$$

$$\kappa = 8.422(28) - 0.0174(10) \cdot (T - 273.15) \quad (5b)$$

$$\sigma_\kappa \cong [(0.014)^2 + (0.0010 \Delta \theta)^2]^{1/2} \quad (5b')$$

where the numbers in parentheses adjoining the coefficients are the numerical values of the standard uncertainties referred to the corresponding last digits.

## 3.2. Analyses and Evaluations of Uncertainty

The uncertainty in the thermal conductivity can be evaluated as [3]

$$\begin{aligned} \frac{\delta \lambda}{\lambda} &= \frac{\delta Q_0}{Q_0} + \frac{\delta(dR_W/dT)}{dR_W/dT} + \frac{\delta(\Delta T_f - \Delta T_i)}{\Delta T_f - \Delta T_i} \\ &+ \frac{\delta \tau \cdot (t_f - t_i)}{t_f \cdot t_i \cdot \ln(t_f/t_i)} + \left( \frac{\delta \lambda}{\lambda} \right)_{rad} + \left( \frac{\delta \lambda}{\lambda} \right)_{pot} + \left( \frac{\delta \lambda}{\lambda} \right)_{sample} \end{aligned} \quad (6)$$

where  $\delta^*$  is the uncertainty of the term (\*),  $R_W$  the resistance of the hot-wire sensor, and  $\delta\tau$  the ambiguity of the time lag of both trigger signals to the bridge and the digital voltmeter. The wall effect can be neglected under the present experimental condition where  $t_f = 1$  s and  $b \cong 3$  mm ( $b$ , radius of the cylindrical wall).

And for thermal diffusivity,

$$\delta\kappa/\kappa = 2 \cdot (\delta a/a) + \delta(\Delta T)_B/(Q_0/4\pi\lambda) + (\delta\lambda/\lambda) \ln(4\kappa 1s/a^2 C) \quad (7)$$

In terms of standard uncertainty [9], the uncertainty of the first term on the right-hand side of Eq. (6) is estimated to be 0.17%, the second term 0.05%, the third term 0.1%, the fourth term 0.05%, the fifth term 0.1%, the sixth term 0.04% (although the potential lead correction was made by 0.08% [3]), and the seventh term about 0.05%, respectively. Finally, the overall accuracy of the measurement of the thermal conductivity is evaluated to be 0.24% as the root-sum-square of subcomponents.

On the other hand, the uncertainty of the first term in Eq. (7) is 0.4%, the second term 1% [ $\delta(\Delta T)_B$ , the initially remaining deflection, in terms

of temperature, from the lack of equilibrium in the bridge], and the third term 1.8% [ $\ln(4\kappa^1 s/a^2 C) \cong 7.5$ ], respectively. Overall the accuracy for thermal diffusivity is 2.1%.

#### 4. COMPARISON WITH OTHER DATA AND DISCUSSION

Figures 2 and 4 show that measured thermal conductivity data are consistent within the claimed uncertainty with the IUPAC recommended formulas and also others [2, 4–6].

The results for the thermal diffusivity have not been compared with other data, because of the lack of reliable data. Therefore, the consistency of the results with the other reference data at 25°C was investigated by comparing the heat capacity  $c$ 's obtained by the formula  $\kappa = \lambda/c\rho$  with literature values [10, 11], as shown in Table III. The densities of the samples used in the measurements were determined with pycnometry. The results (including densities values) agree very well with those presented in the literature [10–12]. The temperature dependence of  $\lambda$  and  $\kappa$  is also consistent with the reference data or the other results.

In conclusion, the accuracy of the empirical equations is estimated to be 0.4 to 0.5% for the thermal conductivity and 4% for the thermal diffusivity (with a coverage factor  $k_p = 2$  (i.e.  $p = 95\%$ )).

**Table III.** Comparison of the Present Data at 25°C with Other Reference Data

	Toluene	<i>n</i> -Heptane
Present results		
$\lambda$ ( $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ )	0.13088	0.12152
$\kappa$ ( $10^{-8} \text{ m}^2 \cdot \text{s}^{-1}$ )	8.972	7.988
$\rho$ ( $10^3 \text{ kg} \cdot \text{m}^{-3}$ )	0.8622	0.6797
$c$ ( $\text{kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ )	1.692	2.238
$(1/\lambda)(d\lambda/dT)$ ( $\text{K}^{-1}$ )	$-2.35 \times 10^{-3}$	$-2.49 \times 10^{-3}$
$(1/\kappa)(dk/dT)$ ( $\text{K}^{-1}$ )	$-3.38 \times 10^{-3}$	$-2.17 \times 10^{-3}$
Other reference data		
IUPAC value [4], $\lambda$	0.1311	0.1228
IUPAC, $(1/\lambda)(d\lambda/dT)$	$-2.287 \times 10^{-3}$	$-2.447 \times 10^{-3}$
TRPC data [10], $c$	1.718	2.245
Kagaku Binran [11], $c$	1.76	2.2429
TRPC [10], $(1/c)(dc/dT)$	$1.885 \times 10^{-3}$	$1.620 \times 10^{-3}$
Phys. Sci. Data [12], $\rho$	0.8623	0.6795
<i>Ibid.</i> [12], $(1/\rho)(dp/dT)$	$-1.096 \times 10^{-3}$	$-1.259 \times 10^{-3}$
Derived, $(1/\kappa)(dk/dT)$	$-3.14 \times 10^{-3a}$	$-2.85 \times 10^{-3a}$

<sup>a</sup> Derived from temperature dependence of specific heat capacity [10] and density [12] and experimental values  $(1/\lambda)(d\lambda/dT)$ .

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